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RADIATION TRANSPORT IN DUST IN DISK GEOMETRY

FINAL TECHNICAL REPORT

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The main objective of the research program is twofold:

- a) to develop a computer code to solve the problem of scattering, absorption and emission of photons by dust grains in a dusty medium with two-dimensional (2-D) disk geometry, and
- b) to study the various physical and geometrical effects of 2-D radiation transport on the thermal structure and radiation field.

As indicated below these tasks have been accomplished. The method for solving the radiation transport problem in disk geometry is a generalization of the quasi-diffusion method (QDM) previously developed by the Principal Investigator.

In this report we shall only give a brief summary of the major results of the research. Detailed discussions can be found in publications resulted from this project (see listing at the end of this report). These results can be divided into four areas as outlined below

### (1) Radiation Transport Code for Disk Geometry

To check the feasibility of applying the QDM of Leung (1975, 1976) to 2-D disk geometries, we first applied the QDM to one-dimensional (1-D) cylindrical geometry which differs from the 2-D disk geometry only in having no transport in the z-direction. Both geometries involve 2 angles. Results indicate that the QDM in this case is stable and rapidly convergent. The result of this effort is a computer code which can solve the 1-D radiation transport problem in three geometries: planar, spherical and cylindrical. The documentation for this code, a generalization of the previous work by Spagna and Leung (1983), is being written up for publication (Egan, Leung and Spagna 1987). Next we generalized the QDM to 2-D geometry and developed a computer code to solve the problem of radiation transport in a dusty medium with disk geometry (Spagna and Leung 1987a). The code can handle arbitrary dust density distribution and other specified local energy source of grain heating (e.g., viscosity, collision with gas component).

In 2-D disk geometry, the basic idea of the QDM, a differential equation method, is to treat the radiative transfer problem as a boundary value problem. The equation of radiation transport is cast into a quasi-diffusion form by combining its zeroth and first angular moment equations through the introduction of several anisotropy factors (components of the anisotropy tensor) which describes the anisotropy of the radiation field. By first assuming the radiation field to be isotropic (diffusion approximation), the system of nonlinear moment and energy-balance equations are linearized and solved to determine the mean intensity and the grain temperature distribution, i.e., the source function. From the source function, we then solve for the angular distribution of the radiation field using a ray-tracing method, to update the anisotropy factors. The coupled nonlinear moment and energy equations are solved by the Newton-Raphson method. Physically, the success of the QDM rests upon the fact that the temperature distribution depends strongly on the radiation energy density and only weakly on the anisotropy of the radiation field. This weak coupling between the thermal structure and radiation field anisotropy allows the problem to be separated into two

distinct steps: solution of the moment and energy-balance equations for a given radiation field anisotropy, and solution of the ray equations to determine the angular distribution of the radiation field for a given source function.

We have used this computer code to study the physics of 2-D radiative transfer in various simple problems (Spagna 1986; Spagna and Leung 1985, 1986, 1987b), as outlined below.

## (2) Effects of Source Geometry

Under the assumption of gray (frequency-independent) opacity, we have compared results from 2-D disk models with those of 1-D geometries (e.g., slab, sphere and infinite cylinder), and have established the range of conditions under which approximation by 1-D geometries may be valid (Spagna and Leung 1984):

- i) thin-disk approximation by 1-D slab geometry when  $Z \ll R$ ;
  - ii) long-cylinder approximation by 1-D cylindrical geometry when  $Z \gg R$ ;
  - iii) thick-disk approximation by 1-D spherical geometry when  $Z \simeq R$ .
- Results indicate that in general the dominant geometry is the one describing the surface closest in optical depth. Here  $R$  is the disk radius and  $Z$  is the half-thickness of the disk. Identifying these conditions allows us to utilize existing codes for 1-D geometries in future calculations when these conditions are met. In addition, we have investigated the feasibility of the following hybrid approach which is intermediate between an exact solution and one based on the diffusion approximation. Instead of solving the ray equations to determine the anisotropy factors self-consistently, we simply use some analytic expressions for these factors, the choice of expressions being guided by results from exact solutions. This drastically reduces the computational effort since for 2-D disk geometry the requirement of computer time is governed by the solution of the ray equations. For some applications, such as those discussed below, this approximate approach gives results which are fairly accurate (to within 20% of the exact solution).

## (3) Application to Externally Heated Dust Clouds

We have studied the various physical and geometrical effects of 2-D radiative transfer on the thermal structure and radiation field anisotropy. In particular, we have computed a set of *nongray* models (using realistic grain opacities) for interstellar dust clouds which are heated externally by the ambient interstellar radiation field. To illustrate one interesting result of this study, here we show results from models with spherical and disk geometries. Both models have the same radius (taken to be 1 parsec) and central optical depth (equal to 100 at  $0.55 \mu\text{m}$ ), the disk model having a 1:1 aspect ratio (ratio of radius to half-thickness,  $R:Z$ ). While the dust temperature distributions in the two cases are very similar as shown in Figure 1, the excess flux for the disk model depends sensitively on the viewing angle  $\theta$  (see Figure 2). The excess flux is defined as the emergent flux minus the flux from the ambient radiation as measured by an observer when the cloud is not resolved. The results in Figure 2 are somewhat unexpected. For clouds

which are unresolved, one would naively expect, since the thermal emission is isotropic in the neighborhood of an emitting grain and the emission (in the far infrared) is optically thin, that the observed flux spectrum should be characteristic only of the dust temperature and independent of viewing angle. However, because of the lack of spherical symmetry in disk geometry and the resulting radiation anisotropy, this is not the case for disk geometry, as shown in Figure 2.

There are essentially two contributions to the variation in observed flux with viewing angle. The first is a purely geometric projection effect. For a disk of radius  $R$ , half-thickness  $Z$ , and having a uniform surface brightness  $I$ , the observed flux as seen by an observer at distance  $D$  ( $D \gg R$  or  $Z$ ) depends on  $\theta$  as

$$F_{\text{disk}} = (\pi R^2 \cos\theta + 4 RZ \sin\theta) \frac{I}{D^2}.$$

This implies that for a disk with a 1:1 aspect ratio, the observed flux is larger seen edge-on than face-on and it reaches a maximum when  $\theta \simeq 52^\circ$ . Compared to a sphere with the same uniform surface brightness and radius, the ratio of observed fluxes for the two geometries is just given by the ratio of their projected areas as seen by the distant observer

$$F_{\text{disk}}/F_{\text{sphere}} = (\pi R^2 \cos\theta + 4 RZ \sin\theta) / \pi R^2,$$

which indicates that for  $R = Z$ ,  $F_{\text{disk}} \geq F_{\text{sphere}}$  for all viewing angles. The second contribution to the variation comes from radiative transfer effects of disk geometry, which affect the radiation field anisotropy, the dust temperature distributions, and hence the surface brightness. For example, for 1-D spherical geometry, energy transport occurs only along a radial direction so that the condition for flux conservation takes a simple form:  $r^2 F(r) = \text{constant}$ , where  $F$  is the frequency integrated net flux. For 2-D disk geometry, the flux conservation equation is more complicated since there is energy transport in both  $r$  and  $z$  directions and in general  $F(r, z)$  depends on both  $r$  and  $z$ . This will significantly affect the radiation field anisotropy. The relative importance of these two contributions depends on such factors as the degree of disk flattening, the cloud optical depth, and the dust density distribution. The importance of the second contribution is supported by the fact that the same trend (though less drastic) as indicated in Figure 2 still persists even when the projection effect is removed by defining an "equivalent flux" (which, in the limit of uniform surface brightness, is independent of viewing angle).

An important consequence of this variation in excess flux with viewing angle is that it implies a large uncertainty in estimating the radiating dust mass for disk-shaped clouds which are unresolved. In particular, *if spherical geometry is incorrectly assumed, the estimated dust mass may easily be off by over an order of magnitude.* Compared to externally heated clouds, the radiation field in clouds with embedded heat source is expected to be more

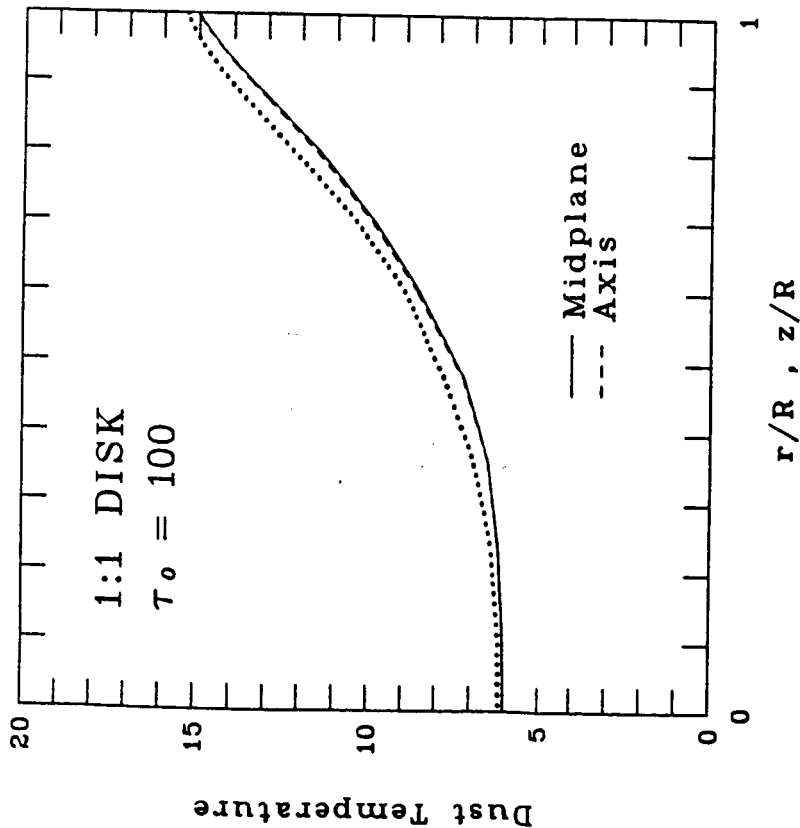


FIGURE 1 Comparison of dust temperature distributions for graphite grains in spherical and disk (with a 1:1 aspect ratio) models of the same central optical depth ( $\tau = 100$ ). The dotted line denotes the radial temperature distribution for the spherical model while the dashed and solid lines denote respectively the temperature profiles along the symmetry axis and along the midplane for the disk model. The three temperature distributions are very similar.

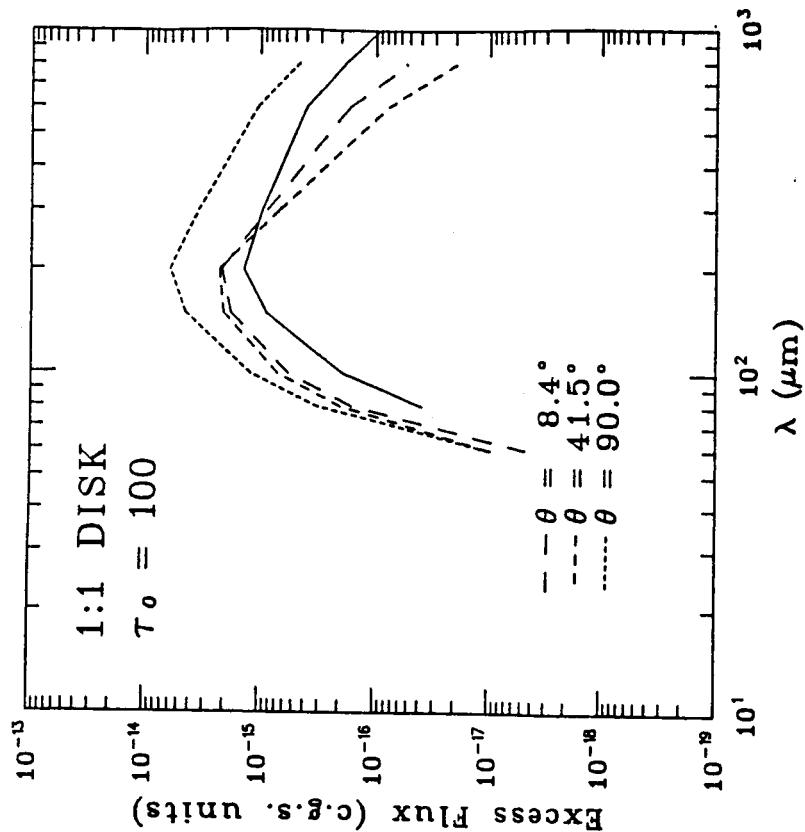


FIGURE 2 Comparison of excess flux spectra for the spherical and disk models of FIGURE 1. The solid line is for the spherically symmetric model. For the disk model, because of the lack of complete symmetry and the resulting radiation anisotropy, the excess flux profile depends on the viewing angle  $\theta$  with respect to the symmetry axis. For edge-on viewing ( $\theta = 90^\circ$ ) the excess flux is strongest.

anisotropic. This implies that the effects of disk geometry on the emergent flux as a function of viewing angle will be even more drastic. Other physical effects of radiation transport in disk geometry and further observational implications are discussed in Spagna and Leung (1986, 1987b).

#### (4) Limb-Brightening Effects in Dark Globules

We have studied the problem of infrared emission from dark globules (e.g., B335, B227) which are nearby dense interstellar clouds that show up as dark patches of obscuration against the general background of stars. A recent review on this subject is given by Leung (1985). If a dark globule contains no embedded heat source and is only heated externally by the interstellar radiation field, it should exhibit limb-brightening in the far infrared (Spencer and Leung 1978). On the other hand, a globule which is internally heated (e.g., by an embedded protostellar object) should exhibit limb-darkening. If the embedded heat source is not strong enough, there may still be limb-brightening. By determining the ratio of intensities at the limb and core, i.e., the limb-brightening ratio (LBR), one can estimate the luminosity of the embedded source. Using realistic cloud models and assuming spherical geometry, we determined these criteria more quantitatively and arrived at the following conclusions:

- a) Limb-brightening effects in the far infrared could be used to set an upper limit on the luminosity of a central heat source embedded in a dark globule.
- b) The threshold wavelength for limb-brightening (the longest wavelength for which LBR exceeds 1.2) is related linearly to the luminosity of the central heat source.
- c) In the absence of a central heat source of significant luminosity, the LBR is related to the optical depth  $\tau$  by a power-law relation

$$\text{LBR} \propto \tau^k, \quad k \approx 0.3$$

in the wavelength region  $120 \mu\text{m} \leq \lambda \leq 240 \mu\text{m}$  where thermal emission from grains peaks (Figure 3).

Thus limb-brightening effect in globules can be used as a diagnostic tool for probing these objects and any undetected embedded sources. In particular, the last conclusion implies that we now have an observational probe for determining the far infrared properties of dust grains in dark globules, i.e., by observing the LBR at various wavelengths within the specified spectral range, the power-law relation can be exploited to determine the emissivity law of the dust grains in the far infrared. However, to determine the LBR accurately, observation at high angular resolution is required. Detailed discussions and observational implications can be found in Leung, Dubisch and O'Brien (1987).

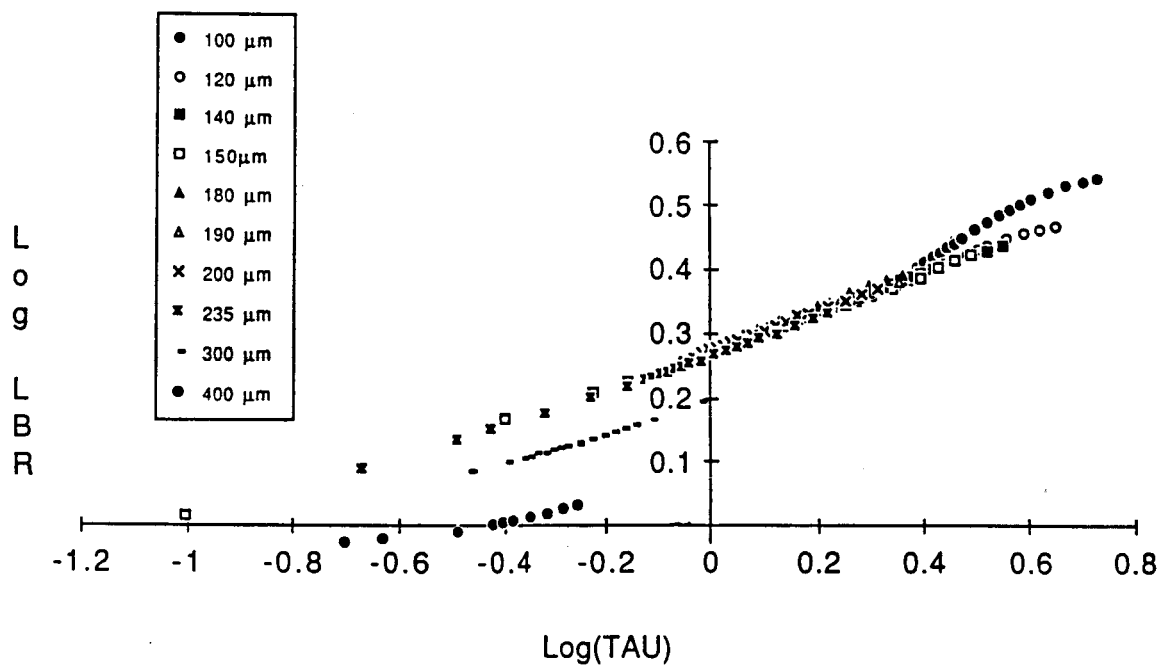


FIGURE 3 Limb-brightening ratio (LBR) vs optical depth (TAU) at various wavelengths for a model of dark globule. For wavelengths (120 - 240  $\mu\text{m}$ ) at which thermal emission from grains mostly occurs, the quantities LBR and TAU satisfy a power-law relation which can be exploited to determine observationally the emissivity law (opacity vs wavelength) in the far infrared of dust grains in dark globules.

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**A. Paper Published**

Spagna, G. F., Jr., and Leung, C. M., "Radiative Transfer in Disk Geometry: Emergent Intensity from Cool Gray Disks.", *Icarus*, 61, 27 (1985).

**B. Paper in Press**

Spagna, G. F., Jr., and Leung, C. M., "Numerical Solution of the Radiation Transport Equation in Disk Geometry.", to appear in the *Journal of Quantitative Spectroscopy and Radiative Transfer*.

**C. Papers in Preparation**

1. Egan, M. P., Leung, C. M., and Spagna, G. F., Jr., "CSDUST3: A Radiation Transport Code for a Dusty Medium with 1-D Planar, Spherical or Cylindrical Geometry.", to be submitted to *Computer Physics Communications*.
2. Leung, C. M., Dubisch, R., and O'Brien, E., "On the Use of Limb-Brightening Effects to Probe Isolated Dark Globules.", to be submitted to the *Astrophysical Journal*.
3. Spagna, G. F., Jr., and Leung, C. M., "Radiation Transport in Disk Geometry. I. Application to Externally Heated Interstellar Clouds.", to be submitted to the *Astrophysical Journal*.

**D. Contributed Papers/Talks Presented in AAS Meetings/Other Conferences**

1. Spagna, G. F., Jr., and Leung, C. M., "Radiative Transport in Disk Geometry: Thermal Structure of Dust Clouds.", *Bull. Amer. Astron. Soc.*, 14, 968 (1982).
2. Spagna, G. F., Jr., and Leung, C. M., "Radiative Transfer in Disk Geometry: Emergent Intensity.", *Bull. Amer. Astron. Soc.*, 15, 930 (1983).
3. Spagna, G. F., Jr., and Leung, C. M., "Effects of Source Geometry on Continuum Radiation Transport.", *Bull. Amer. Astron. Soc.*, 16, 961 (1984).
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5. Dubisch, R., and Leung, C. M., "Modeling of Infrared Emission from Dark Clouds.", talk presented in the ASNY (Astronomical Society of New York) meeting in October 1985.
6. Spagna, G. F., Jr., and Leung, C. M., "Modeling of Infrared Flux Spectra from Disk-Shaped Interstellar Dust Clouds.", poster paper presented in the "Summer School on Interstellar Processes." held in Wyoming in July 1986.

**E. Doctoral Dissertation Completed**

Spagna, G. F., Jr., "Numerical Calculations of Radiation Transport in a Dusty Medium in Disk Geometry.", unpublished Ph. D. Thesis, RPI, May 1986.